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Book Reviews

Finite Difference Methods for Partial Differential Equations. By GEORGE E. FORSYTHE AND WOLFGANG R. WASOW. Wiley, New York, 1960. 444 pp., $11.50.

It is quite gratifying that the numerical analyst has now available to him another work on the subject of finite difference methods for partial differential equations, in addition to that of Richtmyer. The present text represents a very real achievement in exposition in that the book is clearly at a level where even a junior student can understand large portions and benefit thereby. This is achieved in such a fashion, however, that even a specialist will find much of interest.

The subject itself is, of course, a very modern one, dating from the now classical paper of Courant, Friedrichs, and Loewy in the 1928 issue of the *Mathematische Annalen*. Since that time an extensive literature has been built up—the process still continues, perhaps at an accelerated pace—to the point where the present book, which summarizes much of the existing work on the subject, represents almost essential reading for the student of numerical analysis.

The main topics treated are the conventional ones: hyperbolic equations in two independent variables, parabolic equations, elliptic equations, and initial-value problems in more than two independent variables. The authors have taken considerable pains to make the material as useful as possible, including such unusual topics as how to solve elliptic partial difference equations on an automatic digital computer. Material such as this is very valuable and helps to make this book the good one that it is.

In general the authors have been very careful and precise in their discussions; however, the reviewer feels that the introductory discussion of numerical stability is not precise enough and that it is not up to the rest of the text in clarity. In particular, no mention is made of the role of the so-called perturbation equations in stability analyses nor is the definition of stability as crisp as it might be. Also there is no definitive discussion in that place of the relationship between convergence and stability.

With the exception of a few minor points such as this the book is a valuable addition to the literature on numerical analysis. The authors are able to bring together in this place a large number of related topics with a mathematically penetrating point of view. It is well worth while for both students and practitioners of the subject to have this work on their shelves. They will find also that the book is pleasant reading from a typographical point of view.

HERMAN H. GOLDSTINE
I.B.M. Research Center
Yorktown Heights, N. Y.

Information Theory, Statistical Decision Functions, Random Processes. Transactions of the First Prague Conference, held in 1956. Edited by ANTONIN

SPACEK. Publishing House of the Czechoslovak Academy of Sciences, 1957. 354 pp. Transactions of the Second Prague Conference, held in 1959. Edited by ANTONIN SPACEK. Publishing House of the Czechoslovak Academy of Sciences, 1960. 843 pp. Distributed by Academic Press, New York and London.

The ambition of the Prague conferences seems to be to emulate the Berkeley Symposia on Mathematical Statistics and Probability. There is however a big difference of great interest to the readers of this Journal (not counting the obvious fact that the majority of contributors to these meetings are from Eastern Europe): the school of probability and statistics which was created in Prague after the war and which is led by Professor Antonin Spacek, was, interestingly enough, started as a branch of the Institute of Radio Engineering and Electronics of the Czechoslovak Academy of Sciences and is now a branch of the Institute of Information Theory and of Automation. As a result, and in accordance with the titles of the proceedings, these two volumes contain a very high percentage of articles on information theory and on stochastic problems suggested by physical applications. However, the style of these papers follows the more exacting canons of modern mathematics.

The first book is sufficiently old so that we need not review it. The second book is recent and we should have had it reviewed. However, we could not find any person to tackle this impossible job, so that we shall be content with reproducing the (very long) table of contents, referring the reader to *Mathematical Reviews* for detailed examinations of each paper. The lengths of the papers vary from 1 to 150 pages: they are indicated after the titles, in parentheses.

Papers Read at the First Conference

BLACKWELL, D.: The Entropy of Functions of Finite-State Markov Chains

GNEDENKO, B. V.: Some Soviet Work on Information Theory (in Russian)

HANSSON, H.: A Display of Information Theory Problems Concerning Telephone Transmission

RAJSKI, C.: The Selectivity of the Parametric Tests

RAJSKI, C.: The Bayes Rule and the Entropy

PROUZA, L.: Bemerkung zur linearen Prediktion mittels eines lernenden Filters

DRIML, M. AND ŠPAČEK, A.: Continuous Random Decision Processes Controlled by Experience

HANŠ, O.: Generalized Random Variables

HANŠ, O.: Random Fixed Point Theorems

HANŠ, O.: Inverse and Adjoint Transforms of Linear Bounded Random Transforms

HANŠ, O.: Almost Sure Convergence Theorem for Random Schwartz Distributions

NEDOMA, J.: Note on Generalized Random Variables

NEDOMA, J.: The Capacity of a Discrete Channel

PÉREZ, A.: Notions généralisées d'incertitude, d'entropie et d'information du point de vue de la théorie de martingales

PÉREZ, A.: Sur la théorie de l'information dans le cas d'un alphabet abstrait

PÉREZ, A.: Sur la convergence des incertitudes, entropies et informations échantillon (sample) vers leurs valeurs vraies

ŠPAČEK, A.: An Elementary Experience Problem
ŠPAČEK, A.: Prolongement des transformations aléatoires
ULLRICH, M.: Some Theorems on Random Schwartz Distributions
VOTAVOVÁ, L.: Ein Satz von Extremen der Entropie
WINKELBAUER, K.: Experience in Games of Strategy and in Statistical Decision

Papers Read at the Second Conference

BENEŠ, V. E.: Weakly Markov Queues (18)
BHARUCHA-REID, A. T.: On Random Solutions of Integral Equations in Banach Spaces (22)
BREIMAN, L.: Finite-state Channels (12)
DOOB, J. L.: A Relative Limit Theorem for Parabolic Functions (10)
DRIML, M.: Convergence of Compact Measures on Metric Spaces (22)
DRIML, M., AND HANŠ, O.: On Experience Theory Problems (20)
DRIML, M., AND HANŠ, O.: Continuous Stochastic Approximations (10)
DRIML, M., AND HANŠ, O.: Conditional Expectations for Generalized Random Variables (22)
DRIML, M., AND NEDOMA, J.: Stochastic Approximations for Continuous Random Processes (14)
FORTET, R. M.: Problèmes de statistique concernant des processus de Markov (18)
GNEDENKO, B. V.: On a Problem of Queuing Theory (in Russian) (8)
HÁJEK, J.: On a Simple Regression Model in Gaussian Processes (14)
HANŠ, O.: An Elementary Convergence Theorem (4)
HANŠ, O., AND ŠPAČEK, A.: Random Fixed Point Approximation by Differentiable Trajectories (12)
HANSSON, H.: The Entropy of the Swedish Language (4)
HAVEL, J.: An Electronic Generator of Random Sequences (12)
JACOBS, K.: Über die Durchlaßkapazität periodischer und fastperiodischer Kanäle (20)
JAGLOM, A. M. (YAGLOM): Some Formulas on Extrapolation, Filtration and Information in Gaussian Stochastic Processes (in Russian) (2)
KOUTSKÝ, Z.: Einige Eigenschaften der Modulo k addierten Markowschen Ketten (16)
KRICKEBERG, K.: Notwendige Konvergenzbedingungen bei Martingalen und verwandten Prozessen (27)
LAHA, R. G., AND LUKACS, E.: On a Characterization of the Wiener Process (6)
LINNIK, T. B.: On Some Connexions between Shannon's and Fisher's Concepts of Information and the Theory of Addition of Random Vectors (in Russian) (16)
MANDL, P.: Let propriétés limites des répartitions de probabilité des processus de Markov bornés (20)
MARINESCU, G.: Sur les processus stochastiques généralisés (4)
MATTHES, K.: Zur Theorie der Maße in Produkträumen (10)
NEDOMA, J.: On Non-Ergodic Channels (34)
PANTELOPULOS, C.: Processus aléatoires asymptotiquement stationnaires Laplaciens produits par filtrage d'une suite périodique d'impulsions aléatoires (17)

PÉREZ, A.: Sur la théorie de l'information et la discerabilité dans les problèmes de décision statistique (86)
PÉREZ, A.: L'expérience et l'information puisée dans elle a l'aide des lois limites de la théorie des probabilités (22)
PRÉKOPA, A.: On the Spreading Process (10)
PUGACHOV, V. S.: Two Effective Methods of Bayes Decision (in Russian)
RAJSKI, C.: On the Existence of Entropy (2)
RAJSKI, C.: The Pseudo-metric Space of Discrete Random Variables Defined Over a Group (2)
RÉNYI, A.: Dimension, Entropy and Information (12)
RICHTER, H.: Über optimale mehrstufige Tests (12)
ROSENBLATT-ROTH, M.: Normed Epsilon-Entropy and Theory of Transmission of Information (in Russian) (10)
SEIDLER, J.: Relationships Between Information Theory and Decision Function Theory (14)
STATULAVICHUS, V. A.: Some Functionals of Processes (in Russian) (4)
ŠEFL, O.: Filters and Predictors which Adapt Their Values to the Unknown Parameters of the Input Process (12)
ŠPAČEK, A.: Statistical Estimation of Provability in Boolean Logic (18)
ŠPAČEK, A.: Random Metric Spaces (12)
ULLRICH, M.: Random Mikusinski Operators (22)
ULLRICH, M.: Representation Theorem for Random Schwartz Distributions (6)
URBANIK, K.: A Contribution to the Theory of Generalized Stationary Random Fields (14)
VINCZE, I.: An Interpretation of the I-divergence of Information Theory (4)
WINKELBAUER, K.: Communication Channels with Finite Past History (148)
ZIEBA, A.: Fundamental Equations of the Theory of Pursuit (4)
ZÍTEK, F.: Sur certaines propriétés infinitésimales des fonctions aléatoires

BENOIT MANDELBROT
I.B.M. Research Center
Yorktown Heights, N. Y.

Entropy and Capacity of Sets in Function Spaces. By A. N. KOLMOGOROFF AND W. M. TICHOMIROW. Text in German: "Arbeiten zur Informationstheorie, III Entropid und Kapazität von Mengen in Funktionalraümen." Translated by P. FRANKEN AND K. NAWROTZKI from the Russian original which appeared in *Uspekhi Mat. Nauk* **14** (86), 3-86 (1959). Berlin, Deutscher Verlag der Wissenschaften, 1960. 80 pp., price unknown.

L. S. Pontrjagin and L. G. Schnirelman had suggested in 1932 the possibility of characterizing the "concentration" of sets in metric spaces, with the help of the rate of increase with $1/\epsilon$ of the number of elements in the most economic covering of that space with ϵ-neighborhoods. This approach has obvious formal analogies with the problem of construction of error-correcting codes that maximize the number of transmitted "words," for a given minimum "distance" between any two words. Another closely connected question is raised by the problem of the minimization of the numbers of operations that are required for certain numerical algorithms. A third line of investigations concerns the best method of